

An experimental investigation of the streamlines in viscous fluid expelled from a tube

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(Received 24 April 1964)

Experimental confirmation of some streamline patterns suggested by Taylor (1961) is described. It is shown, also, that when a viscous fluid is expelled from a tube by an inviscid fluid, the interface has a 'localized' effect on the fluid ahead of the interface between the two liquids, that is, the usual Hagen–Poiseuille law is obeyed in the viscous fluid except for the region behind a point about $1\frac{1}{2}$ tube diameters ahead of the bubble.

Use of a recently developed tungsten-iodide lamp is also described.

1. Introduction

In 1961, Sir Geoffrey Taylor suggested that the streamlines in a viscous fluid when the fluid is expelled from a tube by an inviscid immiscible fluid would behave in a simple manner, with distinctly different types of behaviour depending on whether m , the fractional amount of fluid left on the wall of the tube, was greater than or less than $\frac{1}{2}$. With his permission, his suggested alternatives are reproduced in figures 1(a), (b), and (c), and are deduced as follows. Assume that there is Poiseuille flow well ahead of the bubble. For a bubble moving with velocity U ,

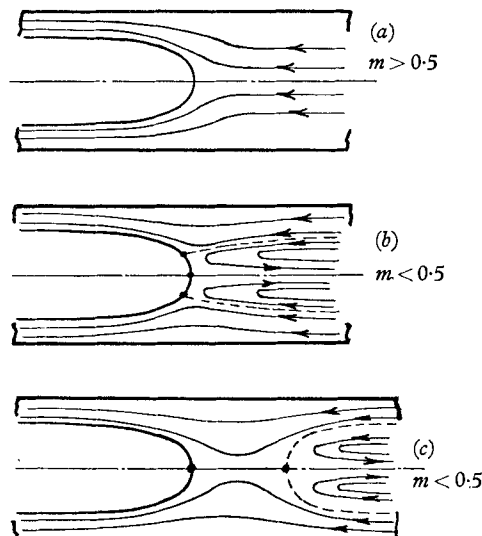


FIGURE 1. Possible patterns of streamlines from Taylor (1961).

superposing an equal and opposite velocity on the system brings the bubble to rest, and the velocity v in the fluid well ahead of the bubble is then given by

$$v = V(1 - r^2/a^2) - U, \quad (1)$$

where V is the velocity on the axis of the tube in a stationary frame of reference, and a is the radius of the tube. But, by continuity of the flow round a bubble of asymptotic radius λa ,

$$2\lambda^2 U = V,$$

so, on the axis of the tube ($r = 0$),

$$v = (2\lambda^2 - 1)U = (1 - 2m)U.$$

Hence, if $m > \frac{1}{2}$, v is negative, and the fluid on the axis is flowing towards the bubble, whereas if $m < \frac{1}{2}$, the fluid is flowing away from the bubble. The wall of the tube is a streamline, so for $m > \frac{1}{2}$ there is only one possible flow as shown in figure 1(a). The situation for $m < \frac{1}{2}$ is very different as reversed flow occurs. Two simple possible situations are shown in figures 1(b) and (c), the former having a stagnation point at the vertex of the bubble, and a stagnation ring on the bubble surface, the latter having two stagnation points on the axis of the tube.

From equation (1), it is obvious that, for $m < \frac{1}{2}$, and well ahead of the bubble, the fluid will be stationary in an annular ring of radius β given by

$$(\beta/a)^2 = 1 - 1/2\lambda^2. \quad (2)$$

Experimental confirmation of figures 1(a) and (b) was obtained by Goldsmith & Mason (1963), as well as other results related to this problem. They did not observe pattern 1(c), however.

Despite the fact that these alternatives had already been investigated, as most of the necessary equipment was to hand, it was thought to be of interest to conduct an experiment to see if a more complicated pattern than 1(b) could be obtained, and to obtain independent confirmation (or otherwise) of pattern 1(a). It was hoped also to obtain experimental data to compare with a numerical solution to be described in a later paper. A more detailed account of the work described below can be found in Cox (1963).

2. Experiment

The apparatus (figure 2) was basically the same as that used for the experiments described in Cox (1962), the only major changes being in the method of photographing the streamlines, and in the method of controlling the velocity of the bubble, by the insertion of a 'resistance' tube in the pressure column. Hence, the pressure column, M , from the reservoir, A , was connected to a tube, F , through which a viscous fluid (pure glycerol) passed with Poiseuille flow, the size and length of the tube being chosen so that the flow in this tube completely dominated the motion in the observation tube. To do this, the pressure column was let into a vessel, D , measuring $4\frac{1}{4}$ in. \times 3 in. \times $2\frac{1}{2}$ in. externally, containing glycerol. This vessel was connected to an identical container, E , by a horizontal brass tube, F , of 1.8 mm bore, and 20 cm length. These two 'resistance' boxes, D and E , had four sides of $\frac{1}{4}$ in. steel plate, 2 in. wide, welded to form an open

rectangle, and the other two sides of $\frac{1}{4}$ in. sheet Perspex, bolted to the steel edges, and sealed by rubber gaskets. Steel was used as both boxes also contained mercury. This seemed to be the easiest way of constructing vessels capable of withstanding a pressure of up to 2 atm., yet allowing visual observation of the mercury-glycerol levels. This method did work successfully in every respect. The remainder of the apparatus was exactly as described in §2 of the above-mentioned paper.

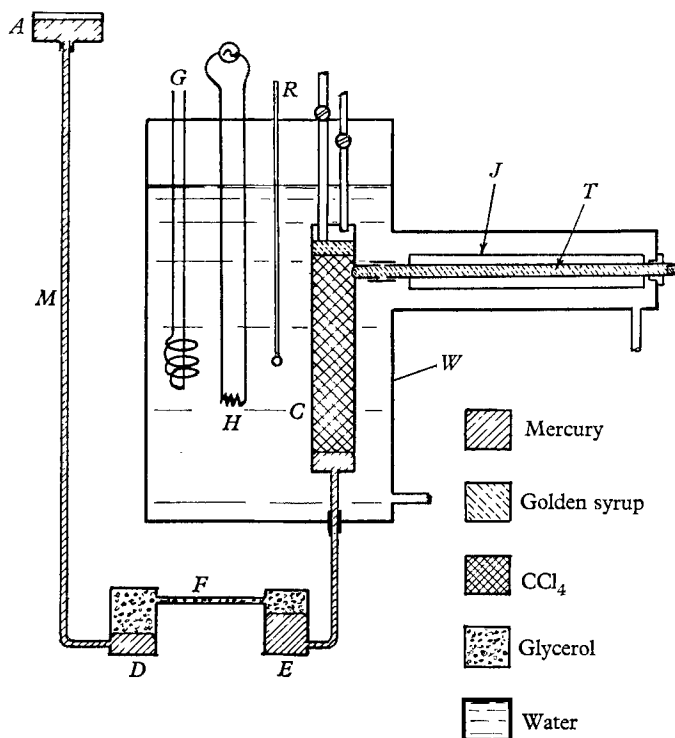


FIGURE 2. Schematic diagram of apparatus.

The second major change in technique was in the photographic method. Tracer particles of finely divided aluminium were added to the Golden syrup before filling the apparatus, and the moving bubble was photographed at 16 frames per second by a Paillard Bolex H-16 16 mm Reflex Ciné Camera, with a 5 mm extension tube on the 25 mm lens, and with the camera moving at the speed of the bubble. To achieve this movement, the camera was mounted on a disused treadle lathe, which had been stripped of all unnecessary weight, and provided with an electric motor geared by a system of cog-wheels to operate as near peak revolutions as possible, so as to give a nearly constant torque. The speed of the motor could still be varied within certain limits by a rheostat connected in compound across the field windings.

Lighting the subject proved much more difficult than expected, and a number of different systems were tried. It was necessary to illuminate the vertical plane containing the axis of the horizontal observation tube, and this was done by

restricting the light to a thin sheet by passing it through two narrow slits, and observing the movement of the tracer particles in this vertical azimuthal plane. For so narrow a sheet, however, most lights commonly used in photography (e.g. photo-floods, or long tungsten filament flood-lights) were singularly unsatisfactory and eventually a recently developed tungsten-iodide bulb of 1500 W power (donated by courtesy of Atlas Lighting Ltd) was found eminently satisfactory. Having a long (25 cm) straight filament, and being a bulb which should be burnt only in a horizontal position, this lamp gave a very even illumination almost the full length of the tube. The considerably higher light efficiency of this new type of bulb permitted the use of a slit as narrow as $\frac{1}{4}$ mm, and yet sharply defined the particles of aluminium in the syrup. To identify the position of the bubble profile, and the position of the wall of the tube, by bringing up the background, a 60 W reading lamp was placed near the camera lens. This lighting system proved quite satisfactory.

Naturally, the choice of the type of film to be used was closely related to the light available. It was found that Kodak Tri-X 16 mm negative film, developed in the laboratory darkroom using the recommended developer, Kodak D-76, with an increase of 50% on the stated developing time to give maximum contrast without sacrificing grain size, produced a film of excellent definition.

The experimental method was much the same as that described in the previous paper. The apparatus was filled, the water-bath topped-up and the thermostat, heater, and circulating pump turned on, and the system left for four or more hours to ensure that it was all at the operating temperature of 30 °C. The focusing and lighting were thoroughly checked, the pressure column reservoir set to the required height, and the experiment started by removing the bung in the end of the tube. The reflex viewing system of the camera made it reasonably easy to match the camera speed to the bubble speed. As the bubbles were not moving very fast (of the order of 0.2 cm/sec), it was usually possible to rewind the mechanism of the camera, and run a second 'take' further down the tube.

The velocity of the bubble was measured by stop-watch, and the width of the bubble and the outer diameter of the tube were measured by projection of the film. Hence λ , the ratio of the bubble width to inner tube width, could be calculated by proportion from the known ratio of the tube diameters to circumvent the known distortion (see Cox 1962), thus providing $m (= 1 - \lambda^2)$, the fractional amount of fluid left in the tube.

The analysis of the ciné-film proved quite difficult, and finally two methods were used in conjunction: a frame-by-frame analysis, and repeated slow motion projection, with the emphasis on the latter. The former method did not give good results largely because the aluminium tracer particles are fine disks, and are only clearly visible if correctly oriented to the light source. This resulted in two effects, which complicate the analysis. First, many particles appeared to pass through the bubble. It was found that these particles were being illuminated by the reading light (which could not conveniently be dispensed with), and their blurred edges showed them to be near the limit of the depth of field of the camera optical system, and hence near the wall of the tube.

The second effect was due to the rotation of the aluminium disks in the shear-

ing flow, and was particularly noticeable in the main region of interest, i.e. near the nose of the bubble, where the streamlines were disturbed from lines parallel to the axis of the tube. This rotation was impossible to trace on a frame-by-frame

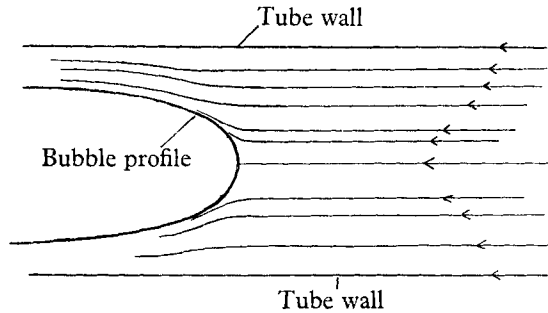


FIGURE 3. Streamline pattern traced from ciné-film; $m = 0.54$.

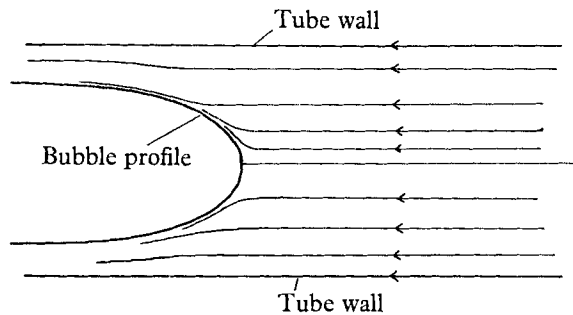


FIGURE 4. Streamline pattern traced from ciné-film; $m = 0.50$.

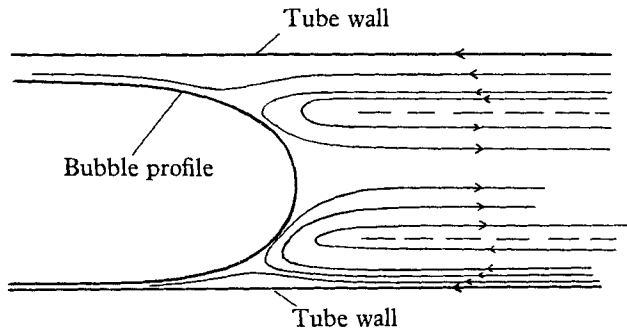


FIGURE 5. Streamline pattern traced from ciné-film; $m = 0.30$.

method, as the only apparatus available for this type of projection was of relatively low power (being primarily designed for a very different purpose), and the particles merged all too easily into the grey background, especially as they changed direction, and so simply disappeared. On the other hand, slow motion projection made the effect readily discernible, as the projector gave a much brighter image, and an almost white background.

The diagrams exhibited above (figures 3, 4 and 5) were found largely by repeated slow motion projection (at least 30 times for each film), and by observing the

motion of the tracer particles in some specific section of the tube. Particular attention was given to where the particles started to deviate from motion parallel to the axis of the tube, and to where they subsequently moved. While this method of analysis is not particularly accurate, it is thought that the resulting diagrams give a good qualitative description of the fluid motion in the cases recorded on film.

For a variety of reasons the system using Golden syrup and CCl_4 described above was not capable of giving a value of m much below 0.50. To obtain a smaller value of m it was necessary to use glycerol as the viscous fluid, and to eject it with a mixture of CCl_4 and White Spirit mixed in the appropriate proportions to give an inviscid fluid of density just greater than glycerol. This still resulted in a slightly non-symmetric interface, but as the densities of the inviscid and the viscous fluids had to be slightly different to allow the apparatus to be filled, a compromise had to be accepted.

The change to glycerol also required the replacement of the syrup in the jacket, J , by glycerol. As the refractive indices of 'Veridia' tubing and glycerol differ only in the third place of decimals, being 1.475 and 1.473, respectively, photographs of this system could be expected to be completely free of distortion. This close match, however, made observation of the tube very difficult as it virtually 'disappeared' into the glycerol—proving the near identity of the refractive indices.

The experiment proceeded substantially as before, but was slightly simplified by discarding the use of the water jacket as the room temperature stayed relatively constant over the period of any one experiment, and the variation of viscosity of glycerol with temperature is substantially smaller than that of Golden syrup. The 1500 W lamp was only left on for short periods to avoid disturbing the ambient temperature, even though the observation tube was largely screened by the two narrow slits from the intense heat radiated by the lamp. Care was taken wherever possible to avoid the absorption of water vapour by glycerol.

3. Discussion of results

(i) *Streamlines*

It is convenient to divide the results into three categories depending on the value of m .

(a) $m > \frac{1}{2}$. Here the results were much as predicted, with no reversed flow. Fluid on the axis well ahead of the bubble moved towards the nose as suggested by the assumption of Poiseuille flow well ahead of the bubble. Figure 3 shows the flow pattern for a typical case, one in which $m = 0.54$. One interesting aspect, however, was the way in which the streamlines were displaced by the bubble. Only streamlines in a core of the tube, the width of this core being of the order of that of the bubble, were significantly displaced, and even these remained virtually undisturbed until they were very near the bubble surface. They then tended to crowd together as they approached the region of uniform flow behind the bubble nose. Hence it seems as though this type of bubble has only a localized effect on the fluid ahead of it. Goldsmith & Mason (1963) also found that the streamlines deviated only when near the bubble.

(b) $m = \frac{1}{2}$. More by chance than design, the first satisfactory film taken gave m approximately equal to $\frac{1}{2}$, and the resulting streamlines were quite interesting, as shown in figure 4. At first it was thought that a central core of the fluid was stationary, but repeated viewing of the film showed that this was not so, and that only the fluid on the axis of the tube could be said to be strictly stationary. Near this line, there was a slight relative motion towards the bubble, as predicted. The streamlines were again deviated round the bubble only when very near the interface. In general, therefore, the streamline pattern was much the same as case (a), and provides further confirmation of the predictions (figure 1) based on simple physical grounds.

It is perhaps worth observing that the above misinterpretation of observation was a result of the natural tendency of the eye to treat the wall of the tube as stationary, and the bubble as moving, instead of vice versa, and the streamline pattern in a stationary frame of reference is, of course, very different from one in which the origin moves with the bubble.

(c) $m < \frac{1}{2}$. It is here that we see the greatest change—and again one qualitatively in accord with the predicted pattern of figure 1(b). Measurement of the profile traced in figure 5 gave $m = 0.30$, and, as expected, a distinct reversed flow was found on the axis, and in a central core about the axis, out to a certain radius where the fluid was stationary. This radius is shown by the broken line in figure 5. Outside this region the fluid flowed backwards towards the bubble, with a slight nip towards a stagnation ring around the nose of the bubble. This ring was very clearly visible on the film. A stagnation point also occurred at the vertex, and particles near the vertex moved very slowly. They achieved, however, a noticeable velocity to the right when only a tenth of a tube diameter ahead of the bubble along the axis. It is apparent from figure 5 that the streamlines outside the stationary layer moved a considerable distance behind the vertex before being reversed.

There was no evidence to support the other possible alternative pattern (figure 1(c)), in accord with the results of Goldsmith & Mason (1963).

It was unfortunate that it was not possible to match the densities of the fluids more closely, and so produce a more symmetric profile, but it is thought that the asymmetry of the profile does not substantially alter the streamline pattern.

As mentioned in the previous section, the method of analysis was not particularly accurate. However, some confidence in the method is felt when the radius of the stationary layer is measured and compared with equation (2). The value of $(\beta/a)^2$ from measurement of figure 5, and the value of $1 - 1/2\lambda^2$ when $\lambda^2 = 0.70$ (i.e. $m = 0.30$), λ being measured from the projected film, are both 0.54. Slow motion projection did in fact show very clearly whether there was, or was not, relative motion between any two points in the fluid, or between a point in the fluid and a point on the interface.

(ii) *Stability*

One aspect that frame-by-frame analysis did show up well was the extreme stability of the bubble interface. This had been observed visually in the experi-

ments described in Cox (1962) and it was amply confirmed by a more detailed comparison possible from the ciné-film. A fully wound camera motor gave approximately 40 sec of recorded observation at 16 frames per second, in which time a bubble moved approximately 8 cm. No departure from the initial bullet-shaped form was apparent in this time. More conclusively, if, as on some occasions, the camera was rewound, and a second length of film exposed when the bubble had advanced much further down the tube, there was again no appreciable difference in the shape of the nose of the bubble in the two sections of the film.

(iii) *Velocity profile*

In each case, analysis of the film showed that the velocity in the fluid about $1\frac{1}{2}$ tube diameters ahead of the bubble was varying with the assumption of Poiseuille flow. This is further evidence in support of the 'localized' effect that the bubble has on the fluid, as suggested in §3(i).

The author is indebted to Sir Geoffrey Taylor for his helpful comments and suggestions in the course of this work, which was carried out during the tenure of a Commonwealth Scholarship.

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